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TO ALL MEMBERS & OTHER INTERESTED PERSONS

Modelling of Yields of Sovereign Bonds
Nelson-Siegel-Svensson Model
Securities Segment

CCIL has been examining the possibility of developing an alternate method for estimating the Sovereign Zero Yield Curve. Nelson-Siegel-Svensson model has been identified as an alternate to the Nelson-Siegel model for constructing the Zero Curve for the Indian Fixed Income Securities market,.

A note containing the suggested method is being put up in CCIL website for information of its members and other interested persons. We would like to seek feedbacks and suggestions from member participants and other interested persons to enable us to adopt a model that is suitable to the Indian Fixed Income Securities market.

Vice President
(Risk Management)

Encl.

Modelling the Yields of Sovereign Bonds – An Alternate Approach

Of the three methods of valuing a Fixed Income Security – Current Yield, YTM and the Coupon, the most common method followed is the Yield To Maturity. The YTM measure is akin to the NPV method of measuring the Hurdle Rate for Long Term Projects. It finds an ***average yield*** that will equate the present value of the periodic cash flows upto the maturity of the security to its cost of procurement. The principle of discounting (compounding) all cash flows at a constant rate implies reinvestment of the coupons also at the *average yield*; which is however not the case in practice. Moreover, the cost of funds as implied by the YTM of a bond will be specific to the bond as it includes the Coupon Effect of the bond; hence, the YTM estimate is not universal in nature.

These demerits of the YTM measure led to the extraction of the yield of every cash flow of the traded bonds as implied by the YTMs of various tenors of the day. This is the Zero Coupon Yield Curve measurement, which identifies the cost (return) of a cash flow of a particular tenor for a bond. Agglomerating the cost (return) of every cash flow of the various bonds traded based on the YTMs gives the ZCYC of the day.

The Time Value of Money obtained from the ZCYC enables valuation of fixed income securities. As per the Expectations Hypothesis, it also indicates the expected future rate of interest for various tenors as implied by the Forward Curve derived from the ZCYC. ZCYC is now used extensively in valuing securities. Traditionally, Central Banks have also been using the ZCYC for gauging the market's expectation of cost of funds in the future.

Commonly adopted methods to extract the ZCYC from the YTM are the Bootstrap and the Parametric form of ZCYC models.

The Bootstrap technique sequentially computes the Zero rate of each cash flow of bonds ordered based on their term to expiry. A curve is fitted through the Zero rates so obtained using line or spline (Cubic Spline is commonly employed).

In case of a parametric form of curve, a pre-decided equation form is fitted to the data and the parameters of the equation form is estimated. Such a curve is easier to implement, enables valuation of any cash flow / bond, and is amenable to simulation for computation of Value-at-Risk.
Considering that very small number of securities are traded in the Indian market and as these do not represent all segments of the curve, CCIL adopted the parametric function to extract the Zero rates from the traded securities of the day. CCIL developed its own Zero Coupon Yield Curve in Sept'02 and started using the same for valuation of the outstanding trade positions of its members in the Securities Settlement Segment. The ZCYC was constructed using the Nelson-Siegel model developed by Charles Nelson and Andrew Siegel of the University of Washington in 1987. This is a parametric function with four parameters for estimating the Forward Rate.

$$f(m,\beta) = \beta_0 + \beta_1 \times \exp(-\frac{m}{\tau_1}) + \beta_2 \times \frac{m}{\tau_1} \times \exp(-\frac{m}{\tau_1})$$

The integration of the Forward Rates across a continuum of maturities upto a tenor point yields the Spot Rate function as

$$r = \beta_0 + (\beta_1 + \beta_2) \times (1 - e^{-(m/\tau)}) / (m/\tau) - \beta_2 \times e^{(-m/\tau)}$$

where $\beta_0, \beta_1, \beta_2, \tau$ - are the parameters to be estimated

r - Zero Rate

m - Maturity

These parameters are related to the long run level of interest rates & the short rate, slope of the yield curve and humps in the curve. These parameters are estimated by the OLS method, which tries to reduce the difference between the estimated Model Prices and the actual Traded Prices of the securities.

Some of the characteristics of the model are:

1. The curve takes only one hump (or U-shape) along the entire length
2. The Model can be easily applied for valuation of any fixed income security

The Goodness of Fit of the CCIL ZCYC has ranged between Re. 0.75 to Rs. 2.75 as measured by the Price Error (difference between the Deal Price and the Model Price) of the traded securities of the day. The movement in the CCIL Zero Curve has generally tracked the corresponding movements in the market-implied YTM curve.

CCIL considers all Outright trades of more than Rs. 5 crores for development of the yield curve. The data is screened for any outliers and fed into a mathematical software. Cash flows are laid down for the traded bonds with their respective Cash Flow Dates. A set of parameters are considered as the starting point for generating the Spot Rates for each cash flow date; the Model Price is computed as the sum of the present values of each cash flow for each traded security. The difference between the Deal Price and the Model Price (Price Error) is weighed by the inverse of the Duration (in years). This process is iterated through various sets of Parameters until the sum of the squared weighted price errors attains the lowest possible level for the portfolio of the securities.

CCIL has been disseminating the N-S ZCYC through its website since Feb-15th, 2003. It has gradually found acceptance with the market due to its goodness of fit and its simplicity.

It has however been observed that the change in the level of yields (slope) w.r.t. tenors is not linear i.e. not a constant slope function. Eg.: On 15th Feb, 2005, the 1-10 year YTM Spread was around 1.50% and the 1-30 year Spread was at 2.25% approximately. Being monotonically changing, N-S model can fit only one of the slope components of the market implied curve; the slope (and the curvature) is continued for the rest of the tenors. This usually results in a better fit for some of the securities falling in a particular tenor zone (especially where there is more concentration of trades) while loosing on accuracy for other tenor buckets. On 15th Feb, 2005, the 10-year benchmark security (7.38% 2015) did not have a good fit (price error of Rs. 2.16) in order to achieve a better fit in the longer end (7.5% GS 2034) (price error of Rs. 1.98 in the negative direction). Moreover, the rate of change in the slope of the market implied curve (curvature) is usually different for different tenor zones.

In our quest to improve the estimation process of the Sovereign Zero Curve, we tried bootstrap method. Bootstrapping pre-supposes availability of traded rates at each six-monthly tenor or at least at most of such tenors. The market of sovereign bonds however is very thin where only few securities are traded (the number of traded securities have come down further in the recent period). In a situation like this, the only option to generate a boot strapped curve was to look for polled prices from the market. Polling process, especially when there is no trade at such prices, can throw up some very inaccurate prices and considering this factor, we did not pursue this idea.
We, then, examined the feasibility of generating an yield curve using Nelson-Siegel-Svensson (N-S-S) model. This is an extension of the N-S model with two additional parameters (to be estimated) which can take an additional hump (or U-Shape). The flexibility to take an additional change in the shape of the curve (curvature) helps in bettering the fit for the various tenures. The N-S-S model was proposed by Lars Svensson in his Research Paper for the National Bureau of Economic Research (NBER), Cambridge, in September 1994. The instantaneous forward rate is modeled as

$$f(m,\beta) = \beta_0 + \beta_1 \times \exp(-\frac{m}{\tau_1}) + \beta_2 \times \frac{m}{\tau_1} \times \exp(-\frac{m}{\tau_1}) + \beta_3 \times \frac{m}{\tau_2} \times \exp(-\frac{m}{\tau_2})$$

The curve asymptotes to a constant value β_0 at the long end, and has a value of $(\beta_0 + \beta_1)$ at the short end, with the flexibility of incorporating an additional hump. The Spot Rate function is given as -

$$r = \beta_0 + (\beta_1 + \beta_2) \times (1 - e^{-(m/\tau_1)})/(m/\tau_1) - \beta_2 \times e^{(-m/\tau_1)} + \beta_3 \times (1 - e^{(-m/\tau_2)})/(m/\tau_2) - \beta_3 \times e^{(-m/\tau_2)}$$

where β_3, τ_2 - are the additional parameters to incorporate an additional slope change and a hump.

This form of the yield curve has been put to use by the Central Banks of some of the advanced countries and we also thought that considering the kind of twist now being seen in the yield curve, it may be worthwhile to develop an Indian Sovereign yield curve on the similar lines, which will have the ability to capture twists in a better manner.

ZCYC using the N-S-S model was initially estimated by us for the period 01st Feb 2005 to 28th Feb 2005. We observed that this form of yield curve was able to fit the data better although the yield slopes were not uniform during this period across the tenors. We therefore examined as to whether N-S-S based equation can provide equally good fit as N-S based equation under normal market condition (i.e. where slope of the yield spreads between various tenors are more or less of the similar order). We examined the quality of fit of N-S-S based equation on data for normal period (from 01st Nov to 30th Nov, 2005) using N-S-S equations. The two periods have different interest rate regimes as measured by the 1-day Rate and the 30 year Zero rate; the 1-day zero rate ranged between 4.50 and 4.50% in Feb-05 and between 5.00 & 5.40% in Nov-04; the 30-year zero rate ranged between 7.00 and 7.5% in Feb-05 and between 7.75 & 8.20% in Nov-04.
A comparison of the ZCYC built by the N-S and N-S-S models for these two periods are placed in **Annexure 1**.

a. The Analysis of the price error and per trade error revealed that the N-S-S model gave a considerably better fit than the N-S model. The N-S-S model gave a Per Trade Error of Re. 0.10 to Re. 0.50 while the N-S model had an error ranging between Re. 0.30 and Re. 1.00. The maximum and the minimum price errors of the various days under test were also lower for the N-S-S model compared to the N-S model; the dispersions were of the order of between Rs. 2.00 and Rs. 3.80 for the N-S-S as against Rs. 3.50 and Rs. 5.00 for the N-S model.

b. We also observed a better fit for the 10-year benchmark security (7.38% GS 2015) of about Re. 0.80 to Rs. 1.50 without loosing the fit for the longer tenor securities like 7.50% GS 2034 where the Price Error ranged between Re.0.50 (-ve) and Re. 0.50 (+ve). The N-S-S model was able to fit the two important tenor zones of interest viz. the medium term zone (5 to 15 yrs.) and the long term zone (20 to 30 yrs.).

c. The N-S-S was found to provide a steepness in the Zero Curve for the Shorter tenor points which was not attainable using the N-S model.

It therefore appears that N-S-S equation based ZCYC may provide a much better alternative for creating a yield curve in the current state of the market. As the number of securities traded may not improve much in the immediate future, it appears that the development of a N-S-S based equation for the Indian Sovereign bond market may be a step in the right direction. This would allow creation of a better quality yield curve even with limited number of traded securities. In regard to usage, it is more like N-S curve, only difference being 6 parameters as against 4 parameters for N-S.

Taking this initiative further, CCIL is considering developing the ZCYC based on the N-S-S model also on a daily basis, compute the Model Prices of all outstanding securities, compare the Model Prices and Price Errors of the traded securities so as to test the advantages likely to arise to the valuation system from the use of N-S-S based yield curve. If this form of yield curve is implemented, CCIL will also consider developing a Utility for the members for computation of Model Price of all outstanding securities using the N-S-S yield curve parameters.

CCIL seeks feedbacks and suggestions on the proposed methodology from market participants. Based on the feedback of the market participants, CCIL will consider adopting the N-S-S model of ZCYC for valuation and margining purposes.

Annexure 1

	Per Trade Error		Price Error			Price Error		
	N-S-S Model	N-S Model	N-S Model			N-S-S Model		
Date			Max.	Min.	Range	Max.	Min.	Range
01-02-05	0.2220	0.6557	1.5181	-2.9337	4.4519	1.2906	-1.6665	2.9571
02-02-05	0.1359	0.5485	1.5634	-2.7801	4.3434	1.1491	-1.4822	2.6312
03-02-05	0.1092	0.5813	1.5821	-2.7395	4.3216	0.9713	-1.1050	2.0763
04-02-05	0.1167	0.5948	1.7231	-2.7017	4.4248	0.9635	-1.1310	2.0945
05-02-05	0.1180	0.5629	1.7241	-2.8998	4.6239	0.8897	-1.3414	2.2311
07-02-05	0.3087	0.8825	1.8786	-2.8807	4.7592	1.6996	-1.2411	2.9407
08-02-05	0.3185	0.9197	2.2226	-2.6785	4.9011	1.3083	-2.1056	3.4139
09-02-05	0.2442	0.7538	2.1673	-2.3744	4.5417	1.3852	-1.7065	3.0917
10-02-05	0.3188	0.9037	2.2148	-2.0517	4.2664	1.2751	-2.2775	3.5526
11-02-05	0.4861	0.8918	2.2445	-2.2915	4.5359	2.1499	-2.3953	4.5452
12-02-05	0.3972	0.7980	2.3921	-1.8507	4.2428	1.9499	-1.5911	3.5411
14-02-05	0.3935	0.8640	2.3716	-2.6158	4.9874	1.5121	-1.8984	3.4105
15-02-05	0.3979	0.9842	2.1607	-2.3879	4.5485	1.7440	-1.5437	3.2877
16-02-05	0.2829	0.5702	2.1620	-1.8562	4.0182	1.5742	-1.9037	3.4780
17-02-05	0.5339	0.6072	2.1177	-1.9751	4.0928	0.9359	-2.4898	3.4257
18-02-05	0.3745	0.8964	2.2319	-2.3248	4.5567	1.6348	-1.5720	3.2068
19-02-05	0.2712	0.8320	2.2225	-1.6564	3.8789	0.9921	-1.8837	2.8758
21-02-05	0.2947	0.6200	2.1993	-1.6747	3.8740	1.3839	-2.3403	3.7242
22-02-05	0.3219	0.6260	2.1798	-1.6386	3.8184	1.5303	-1.9691	3.4994
23-02-05	0.3167	0.5349	2.2164	-1.4202	3.6365	1.5319	-1.5286	3.0605
24-02-05	0.2775	0.4955	2.2271	-1.3590	3.5861	1.7637	-1.3195	3.0832
25-02-05	0.2026	0.3617	2.2500	-1.4944	3.7443	1.5059	-1.4201	2.9260
26-02-05	0.3801	0.5796	2.2344	-1.6254	3.8598	1.4436	-2.1322	3.5758
28-02-05	0.2689	0.5896	2.0783	-1.7576	3.8358	1.5041	-1.2851	2.7891

	Per Trade Error		Price Error			Price Error		
	N-S-S Model	N-S Model	N-S Model			N-S-S Model		
Date			Max.	Min.	Range	Max.	Min.	Range
01-11-05	0.1805	1.0117	1.0158	-2.2836	3.2993	1.2637	-0.7906	2.0543
02-11-05	0.2058	1.1555	2.1109	-2.1367	4.2476	0.9394	-1.4342	2.3736
03-11-05	0.2452	1.6842	2.0322	-2.8290	4.8612	1.2235	-0.8371	2.0606
04-11-05	0.2320	1.1097	2.4147	-2.4987	4.9134	1.3307	-1.2300	2.5608
05-11-04	0.5016	1.7488	2.6322	-2.6761	5.3083	1.2907	-2.3334	3.6241
06-11-04	0.5992	1.5294	3.8452	-2.0023	5.8476	1.9278	-1.6047	3.5325
08-11-04	0.3554	0.6491	2.3419	-1.6800	4.0219	2.3254	-1.2603	3.5857
09-11-04	0.2225	0.8616	0.9055	-4.0487	4.9542	1.3328	-1.2425	2.5753
10-11-04	0.2078	1.2456	0.9535	-3.4874	4.4409	1.3586	-0.9798	2.3384
11-11-04	0.1194	0.9535	0.5821	-3.2179	3.8000	1.1193	-0.5156	1.6349
13-11-04	0.0015	0.3557	0.5710	-1.1990	1.7699	0.0771	-0.0493	0.1264
16-11-04	0.2646	1.1188	1.2821	-2.3943	3.6764	1.2745	-1.4014	2.6759
17-11-04	0.2697	1.1487	1.2574	-2.3138	3.5712	1.2075	-1.0794	2.2868
18-11-04	0.2378	1.0378	1.3697	-2.4361	3.8059	1.5138	-1.5228	3.0367
19-11-04	0.2762	0.9207	1.1941	-2.5843	3.7783	1.2328	-1.5722	2.8050
20-11-04	0.2157	0.7448	1.3249	-2.5036	3.8284	1.2227	-1.2975	2.5202
22-11-04	0.2938	1.1021	1.2896	-2.6089	3.8985	1.3946	-1.5165	2.9111
23-11-04	0.5051	1.4863	1.1006	-3.0363	4.1368	1.3914	-2.6557	4.0472
24-11-04	0.3031	1.1792	1.1027	-2.7512	3.8539	1.3203	-2.2083	3.5287
25-11-04	0.4730	1.0312	1.1200	-3.0508	4.1708	1.5343	-2.7376	4.2719
27-11-04	0.3146	1.1926	0.2506	-2.8389	3.0895	0.5665	-1.5375	2.1040
29-11-04	0.3161	1.4910	1.0261	-2.5736	3.5998	0.4941	-1.3955	1.8896
30-11-04	0.1949	1.0057	1.0070	-2.2512	3.2581	1.4618	-1.3268	2.7885